Use identities to simplify each expression:

1. $\dfrac{\tan x \csc x}{\sec x} = \dfrac{\left(\dfrac{\sin x}{\cos x}\right)\left(\dfrac{1}{\sin x}\right)}{\dfrac{1}{\cos x}} = \dfrac{\dfrac{1}{\cos x}}{\dfrac{1}{\cos x}} = \left(\dfrac{1}{\cos x}\right)\left(\dfrac{\cos x}{1}\right) = 1$

2. $\tan^2 x - \dfrac{\sin(-x)}{\sin x} = \tan^2 x - \left(\dfrac{-\sin x}{\sin x}\right) = \tan^2 x + 1 = \sec^2 x$

Multiply and simplify:

3. $\sin\theta\cos\theta(\tan\theta + \cot\theta) = \sin\theta\cos\theta\left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right) = \sin^2\theta + \cos^2\theta = 1$

Factor and simplify:

4. $\sin^2 x \tan^2 x + \sin^2 x = \sin^2 x(\tan^2 x + 1) = \sin^2 x(\sec^2 x) = \sin^2 x\left(\dfrac{1}{\cos^2 x}\right) = \tan^2 x$

5. $\dfrac{\sin x \cos x}{\tan x} = 1 - \sin^2 x$

$$\begin{aligned}
\frac{\sin x \cos x}{\tan x} &= \frac{\sin x \cos x}{\dfrac{\sin x}{\cos x}} \\
&= \sin x \cos x\left(\frac{\cos x}{\sin x}\right) \\
&= \cos^2 x \\
&= 1 - \sin^2 x
\end{aligned}$$

6. $\cot(-x) = \dfrac{1-\sin^2 x}{\cos(-x)\sin(-x)}$

$$\begin{aligned}
\frac{1-\sin^2 x}{\cos(-x)\sin(-x)} &= \frac{\cos^2 x}{\cos x(-\sin x)} \\
&= \frac{\cos x}{-\sin x} \\
&= -\cot x
\end{aligned}$$

$\cot(-x) = -\cot x$

7. $\dfrac{\sin 2\beta}{2\csc\beta} = \sin^2\beta\cos\beta$

$$\begin{aligned}
\frac{\sin 2\beta}{2\csc\beta} &= \frac{2\sin\beta\cos\beta}{\dfrac{2}{\sin\beta}} \\
&= 2\sin\beta\cos\beta\left(\frac{\sin\beta}{2}\right) \\
&= \sin^2\beta\cos\beta
\end{aligned}$$

8. $\dfrac{1}{\sec\theta-1}-\dfrac{1}{\sec\theta+1}=2\cot^2\theta$

$$\begin{aligned}
\frac{1}{\sec\theta-1}-\frac{1}{\sec\theta+1}&=\left(\frac{1}{\sec\theta-1}\right)\left(\frac{\sec\theta+1}{\sec\theta+1}\right)-\left(\frac{1}{\sec\theta+1}\right)\left(\frac{\sec\theta-1}{\sec\theta-1}\right)\\
&=\frac{\sec\theta+1-(\sec\theta-1)}{(\sec\theta+1)(\sec\theta-1)}\\
&=\frac{\sec\theta+1-\sec\theta+1}{\sec^2\theta-1}\\
&=\frac{2}{\tan^2\theta}\\
&=2\cot^2\theta
\end{aligned}$$

9. $\cos(3x)=\cos x\left(1-4\sin^2 x\right)$

$$\begin{aligned}
\cos(3x)&=\cos(2x+x)\\
&=\cos 2x\cos x-\sin 2x\sin x\\
&=\left(1-2\sin^2 x\right)\cos x-(2\sin x\cos x)\sin x\\
&=\cos x-2\sin^2 x\cos x-2\sin^2 x\cos x\\
&=\cos x-4\sin^2 x\cos x\\
&=\cos x\left(1-4\sin^2 x\right)
\end{aligned}$$

10. $\sin^2\left(\dfrac{x}{2}\right)=\dfrac{\csc^2 x-\cot^2 x}{2\csc^2 x+2\csc x\cot x}$

$$\begin{aligned}
\frac{\csc^2 x-\cot^2 x}{2\csc^2 x+2\csc x\cot x}&=\frac{1}{\dfrac{2}{\sin^2 x}+\left(\dfrac{2}{\sin x}\right)\left(\dfrac{\cos x}{\sin x}\right)}\\
&=\frac{1}{\dfrac{2+2\cos x}{\sin^2 x}}\\
&=\frac{\sin^2 x}{2+2\cos x}\\
&=\frac{1-\cos^2 x}{2+2\cos x}\\
&=\frac{(1-\cos x)(1+\cos x)}{2(1+\cos x)}\\
&=\frac{1-\cos x}{2}
\end{aligned}$$

$$\begin{aligned}
\sin^2\left(\frac{x}{2}\right)&=\left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2\\
&=\frac{1-\cos x}{2}
\end{aligned}$$

11. Use trigonometric identities to find the values of the other five trigonometric functions if $\cos\alpha = 1/\sqrt{5}$ and α is in quadrant IV.

In quadrant IV, cosine and secant are positive, and all the other functions are negative.

$$\sec\alpha = \frac{1}{\cos\alpha} = \frac{1}{1/\sqrt{5}} = \sqrt{5}$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\sin^2\alpha + \left(\frac{1}{\sqrt{5}}\right)^2 = 1$$

$$\sin^2\alpha + \frac{1}{5} = 1$$

$$\sin^2\alpha = \frac{4}{5}$$

$$\sin\alpha = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}}$$

$$\csc\alpha = \frac{1}{\sin\alpha} = \frac{1}{-2/\sqrt{5}} = -\frac{\sqrt{5}}{2}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-2/\sqrt{5}}{1/\sqrt{5}} = -2$$

$$\cot\alpha = \frac{1}{\tan\alpha} = -\frac{1}{2}$$

Find the exact value by using a sum or difference identity:

12. $\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$$= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

13. $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$$= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

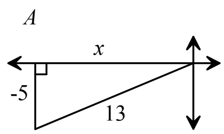
14. $\tan(165^\circ) = \tan(120^\circ + 45^\circ) = \dfrac{\tan(120^\circ) + \tan(45^\circ)}{1 - \tan(120^\circ)\tan(45^\circ)} = \dfrac{-\sqrt{3}+1}{1-(-\sqrt{3})(1)} = \dfrac{-\sqrt{3}+1}{1+\sqrt{3}}$

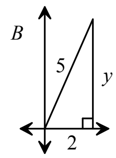
Use the sum/difference identities to simplify each expression.

15. $\cos 75^\circ \cos 60^\circ - \sin 75^\circ \sin 60^\circ = \cos(75^\circ + 60^\circ) = \cos 135^\circ = -\sqrt{2}/2$

16. $\sin 80^\circ \cos 50^\circ - \cos 80^\circ \sin 50^\circ = \sin(80^\circ - 50^\circ) = \sin 30^\circ = 1/2$

17. Find $\sin(A+B)$ if $\sin A = -5/13$ and $\cos B = 2/5$, with A in quadrant III and B in quadrant I.





$$x = -\sqrt{13^2 - (-5)^2} = -12$$

$$\cos A = -\frac{12}{13}$$

$$y = \sqrt{5^2 - 2^2} = \sqrt{21}$$

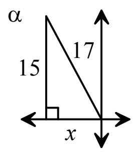
$$\sin B = \frac{\sqrt{21}}{5}$$

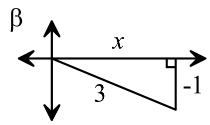
$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \left(-\frac{5}{13}\right)\left(\frac{2}{5}\right) + \left(-\frac{12}{13}\right)\left(\frac{\sqrt{21}}{5}\right)$$

$$= \frac{-10 - 12\sqrt{21}}{65}$$

18. Find $\cos(\alpha - \beta)$ if $\sin\alpha = 15/17$ and $\sin(\beta) = -1/3$, with α in quadrant II and β in quadrant IV.





$$x = -\sqrt{17^2 - 15^2} = -8$$

$$\cos\alpha = -\frac{8}{17}$$

$$x = \sqrt{3^2 - (-1)^2} = \sqrt{8} = 2\sqrt{2}$$

$$\cos\beta = \frac{2\sqrt{2}}{3}$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$= \left(-\frac{8}{17}\right)\left(\frac{2\sqrt{2}}{3}\right) + \left(\frac{15}{17}\right)\left(-\frac{1}{3}\right)$$

$$= \frac{-16\sqrt{2} - 15}{51}$$

Find the exact value by using a half-angle identity.

19. We know $\sin(-\pi/8)$ is negative because $-\pi/8$ is in quadrant IV.

$$\sin(-\pi/8) = \sin\left(\frac{-\pi/4}{2}\right) = -\sqrt{\frac{1-\cos(-\pi/4)}{2}} = -\sqrt{\left(\frac{1-\sqrt{2}/2}{2}\right)\left(\frac{2}{2}\right)} = -\sqrt{\frac{2-\sqrt{2}}{4}} = \frac{-\sqrt{2-\sqrt{2}}}{2}$$

20. We know that $\tan(3\pi/8)$ is positive because $3\pi/8$ is in quadrant I.

$$\tan(3\pi/8) = \tan\left(\frac{3\pi/4}{2}\right)$$

$$= \frac{1-\cos(3\pi/4)}{\sin(3\pi/4)} = \frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{\frac{\sqrt{2}}{2}} = \left(\frac{1+\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\right)\cdot\frac{2}{2} = \frac{2+\sqrt{2}}{\sqrt{2}}$$

$$= \left(\frac{2+\sqrt{2}}{\sqrt{2}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{2\sqrt{2}+2}{2} = \sqrt{2}+1$$

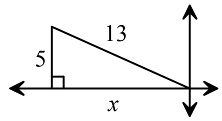
Use the given information to find the exact value of the trigonometric function.

21. Find $\cos\left(\frac{\alpha}{2}\right)$ if $\cos\alpha = \frac{1}{4}$, and α is in Quadrant IV.

$270° < \alpha < 360° \Rightarrow 135° < \alpha/2 < 180°$, so $\alpha/2$ is in quadrant II and $\cos(\alpha/2)$ is negative.

$$\cos\left(\frac{\alpha}{2}\right) = -\sqrt{\frac{1+\cos\alpha}{2}} = -\sqrt{\frac{1+1/4}{2}} = -\sqrt{\frac{5/4}{2}} = -\sqrt{\frac{5}{8}} = -\frac{\sqrt{5}}{2\sqrt{2}}$$

22. Find $\sin(2\theta)$ if $\sin(\theta)=\dfrac{5}{13}$, and θ is in quadrant II.



$$\sin(2\theta)=2\sin\theta\cos\theta=2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right)=-\frac{120}{169}$$

$x=-\sqrt{13^2-5^2}=-12$

$\cos\theta=-\dfrac{12}{13}$

Find the exact value without using a calculator.

23. $\cos^{-1}\left(-\dfrac{1}{2}\right)=\dfrac{2\pi}{3}$ — Remember, $\cos^{-1}$ is between 0 and π.

24. $\arctan(1)=\dfrac{\pi}{4}$ — Remember, arctan is between $-\dfrac{\pi}{2}$ and $\dfrac{\pi}{2}$.

25. $\sec^{-1}\left(\dfrac{2\sqrt{3}}{3}\right)=\sec^{-1}\left(\dfrac{2}{\sqrt{3}}\right)=\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)=\dfrac{\pi}{6}$ — Remember, $\sec^{-1}$ is between 0 and π.

Find the exact value <u>in degrees</u> without using a calculator.

26. $\csc^{-1}\left(-\sqrt{2}\right)=\sin^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)=-45°$ — Remember, $\csc^{-1}$ is between $-90°$ and $90°$.

27. $\arcsin\left(\dfrac{1}{2}\right)=30°$ — Remember, arcsin is between $-90°$ and $90°$.

28. $\cot^{-1}\left(-\sqrt{3}\right)=\cot^{-1}\left(\dfrac{-\sqrt{3}/2}{1/2}\right)=150°$ — Remember, $\cot^{-1}$ is between $0°$ and $180°$.

$\left(\cot x=\dfrac{\cos x}{\sin x},\text{ so }\cos x=-\dfrac{\sqrt{3}}{2}\text{ and }\sin x=\dfrac{1}{2}.\right)$

Find the approximate value using a calculator.

29. $\sec^{-1}(-1.5643)=\cos^{-1}\left(\dfrac{1}{-1.5643}\right)=2.26$

Find all values for x in the interval $[0,2\pi]$ that satisfy the equation.

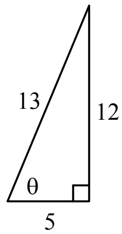
30. $\tan x=\sqrt{3}$

$x=\dfrac{\pi}{3}+k\pi$

$\left\{\dfrac{\pi}{3},\ \dfrac{4\pi}{3}\right\}$

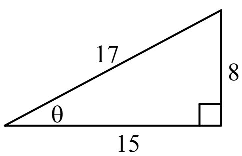
Find the exact value of each composition.

31. $\csc\underbrace{\left(\cos^{-1}\left(\frac{5}{13}\right)\right)}_{\theta}$ What is the cosecant of the angle whose cosine is $\frac{5}{13}$?



$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{13}$ $\text{opp} = \sqrt{13^2 - 5^2} = 12$ $\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{13}{12}$

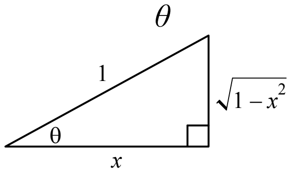
32. $\cos\underbrace{\left(\arctan\left(\frac{8}{15}\right)\right)}_{\theta}$ What is the cosine of the angle whose tangent is $\frac{8}{15}$?



$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{15}$ $\text{hyp} = \sqrt{8^2 + 15^2} = 17$ $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{15}{17}$

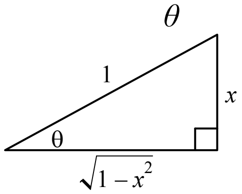
Find an equivalent algebraic expression for each composition.

33. $\tan\underbrace{\left(\arccos\left(x\right)\right)}_{\theta}$ What is the tangent of the angle whose cosine is x?



$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{1}$ $\text{opp} = \sqrt{1^2 - x^2} = \sqrt{1-x^2}$ $\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{1-x^2}}{x}$

34. $\sec\underbrace{\left(\arcsin\left(x\right)\right)}_{\theta}$ What is the secant of the angle whose sine is x?



$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{x}{1}$ $\text{adj} = \sqrt{1^2 - x^2} = \sqrt{1-x^2}$ $\sec x = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\sqrt{1-x^2}}$

Find the <u>acute</u> angle θ, to the nearest hundredth <u>of a degree</u> for the given function value.

35. $\csc\theta = 6.354 \Rightarrow \sin\theta = \frac{1}{6.354}$

$$\theta = \sin^{-1}\left(\frac{1}{6.354}\right) = 9.05^\circ$$

36. $\sec\theta = 4.321 \Rightarrow \cos\theta = \frac{1}{4.321}$

$$\theta = \cos^{-1}\left(\frac{1}{4.321}\right) = 76.62^\circ$$

Find all real numbers that satisfy the equation.

37. $2\cos x + \sqrt{3} = 0$

$$2\cos x = -\sqrt{3}$$

$$\cos x = -\frac{\sqrt{3}}{2}$$

$$\left\{ x \,\middle|\, x = \frac{5\pi}{6} + 2k\pi \text{ or } x = \frac{7\pi}{6} + 2k\pi \right\}$$

Find all real numbers in the interval $[0, 2\pi)$ that satisfy the equation.

38. $\sqrt{2}\sin(3x) = 1$

$$\sin(3x) = \frac{1}{\sqrt{2}}$$

$$3x = \frac{\pi}{4} + 2k\pi \text{ or } 3x = \frac{3\pi}{4} + 2k\pi$$

$$x = \frac{\pi}{12} + \frac{2k\pi}{3} \text{ or } x = \frac{\pi}{4} + \frac{2k\pi}{3}$$

$$\frac{\pi}{12} + \frac{2\pi}{3} = \frac{\pi}{12} + \frac{8\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

$$\frac{\pi}{4} + \frac{2\pi}{3} = \frac{3\pi}{12} + \frac{8\pi}{12} = \frac{11\pi}{12}$$

$$\left\{ \frac{\pi}{12}, \frac{3\pi}{4}, \frac{\pi}{4}, \frac{11\pi}{12} \right\}$$

Find all real numbers that satisfy the equation.

39. $2\cos^2 x = \cos x$

$$2\cos^2 x - \cos x = 0$$

$$\cos x(2\cos x - 1) = 0$$

$$\cos x = 0 \text{ or } 2\cos x - 1 = 0$$

$$\cos x = 0 \qquad \text{or} \qquad \cos x = \frac{1}{2}$$

$$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \text{ etc.} \qquad \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\left\{ x \,\middle|\, x = \frac{\pi}{2} + k\pi \text{ or } x = \frac{\pi}{3} + 2k\pi \text{ or } x = \frac{5\pi}{3} + 2k\pi \right\}$$

Find all angles in the interval $[0°, 360°)$ that satisfy the equation.

40. $2\sin x \cos x = 1$

$$\sin(2x) = 1$$

$$2x = 90° + 360°k$$

$$x = 45° + 180°k$$

$$\{45°, 225°\}$$